Advanced Learners and the new NC SCOS

Advanced Math and Science Content for Teachers of Advanced Learners

MATH SESSION # 2

Using Recursion to Explore Real World Problems Part I

Grades 7 - 12

Date: November 27, 2012

Developed in partnership with DPI~AIG and NC School of Science and Mathematics

Advanced Content for Teachers of Advanced Learners

- Why?
  - To ensure the growth of advanced learners
  - To develop teachers' understanding of advanced math/science content and instructional practices

- What?
  - 14 Content-based PD sessions, webinar and archived; 7 mathematics, 7 science

Using Recursion to Explore Real World Problems Part I

Goals: To introduce the topic of recursion to model real-world scenarios. The webinar's content is focused on Common Core State Standards in Math that are listed on the next slide.

We will explore "big picture" scenario and then use the calculator to generate numerical and graphical representations for a couple problems.

Recursion & NC Standard Course of Study

- F-BF.1. Write a function that describes a relationship between two quantities.
  - Determine an explicit expression, a recursive process, or steps for calculation from a context.
- F-BF.2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
- F-IF.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

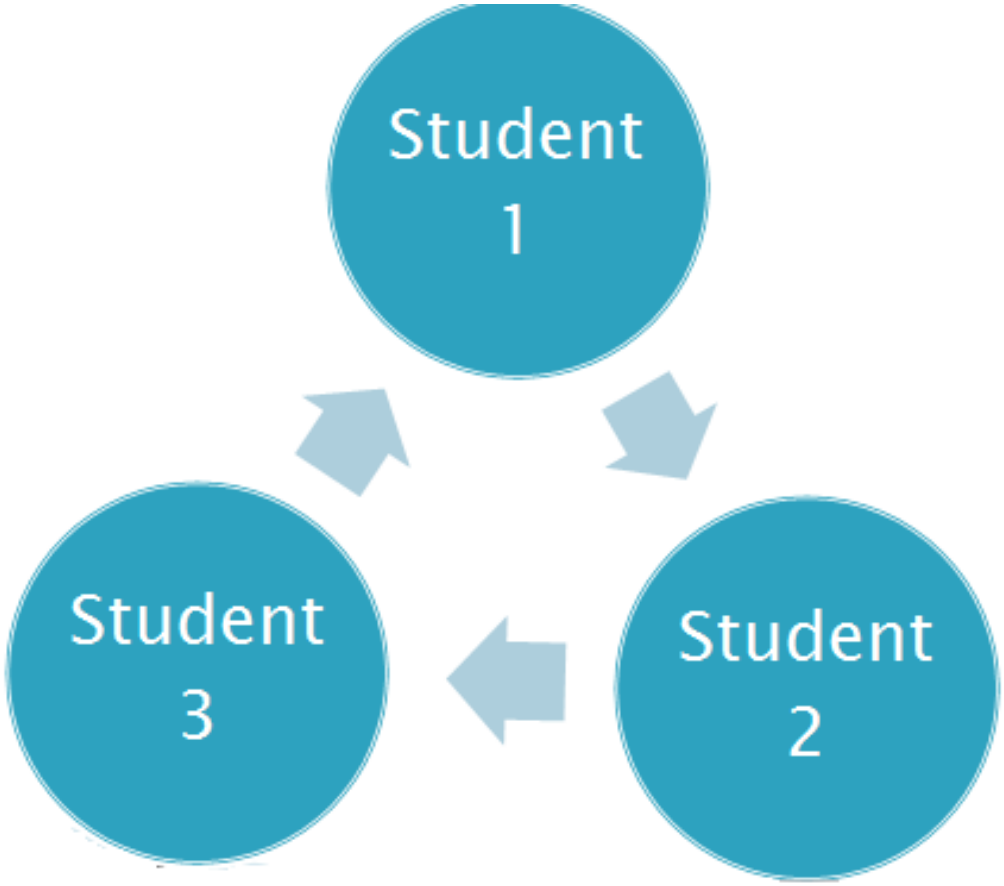
As we work through the problems:

- Consider how your student will approach the problem.
- Anticipate their responses and questions.
- Consider how we can use their ideas and contributions to build the mathematical concepts.

Pass the Candy

- We will form a group or two of 3 people. Appoint a record keeper.
- Each member will get a bag with some candy. **Count** the number of pieces of candy in your bag and tell the record keeper that number so they can record it on the Tally Sheet.
- When I say “Pass the Candy”, we will each pass half of our candy to the person on our left. We will count the number of pieces of candy and record that number on the Tally Sheet.

Student 1 passes to Student 2, Student 2 passes to student 3 and Student 3 Passes to Student 1



Some Questions to Consider…

If we continue this process and start with

Student 1 has 20, Student 2 has 8, Student 3 has 2 (for example)

1. What do YOU predict will happen in the long run?
2. Who will end up with the most? The least?
3. Later: What happens if you change the initial numbers?

Now continue the process and record your results.

Explore the simulation – Python program written by NCSSM Math Faculty

Example 1: Headache Problem

You have a headache and decide to take a 200-mg over-the-counter headache medicine tablet to relieve it. The ibuprofen in the pill is absorbed into your system and stays there until it is metabolized and filtered out by the liver and kidneys. Every 4 hours, your body removes 67% of the ibuprofen that was in your body at the beginning of that 4-hour time period.

How much of the ibuprofen will be in your system 24 hours after taking the tablet?

Using the calculator to answer the question

Type 200 on the home screen

And press ENTER. Then use the answer key to refer to the previous amount of the drug in your system.

How many iterations will we need to answer the question?

Let's write a recursive equation for the scenario – choice of notation:

Function notation

$$D(0) = 200$$

$$D(1) = D(0) - 0.67D(0) = 0.33D(0)$$

$$D(2) = D(1) - 0.67D(1) = 0.33D(1)$$

$$\vdots$$

$$D(n) = 0.33D(n-1)$$

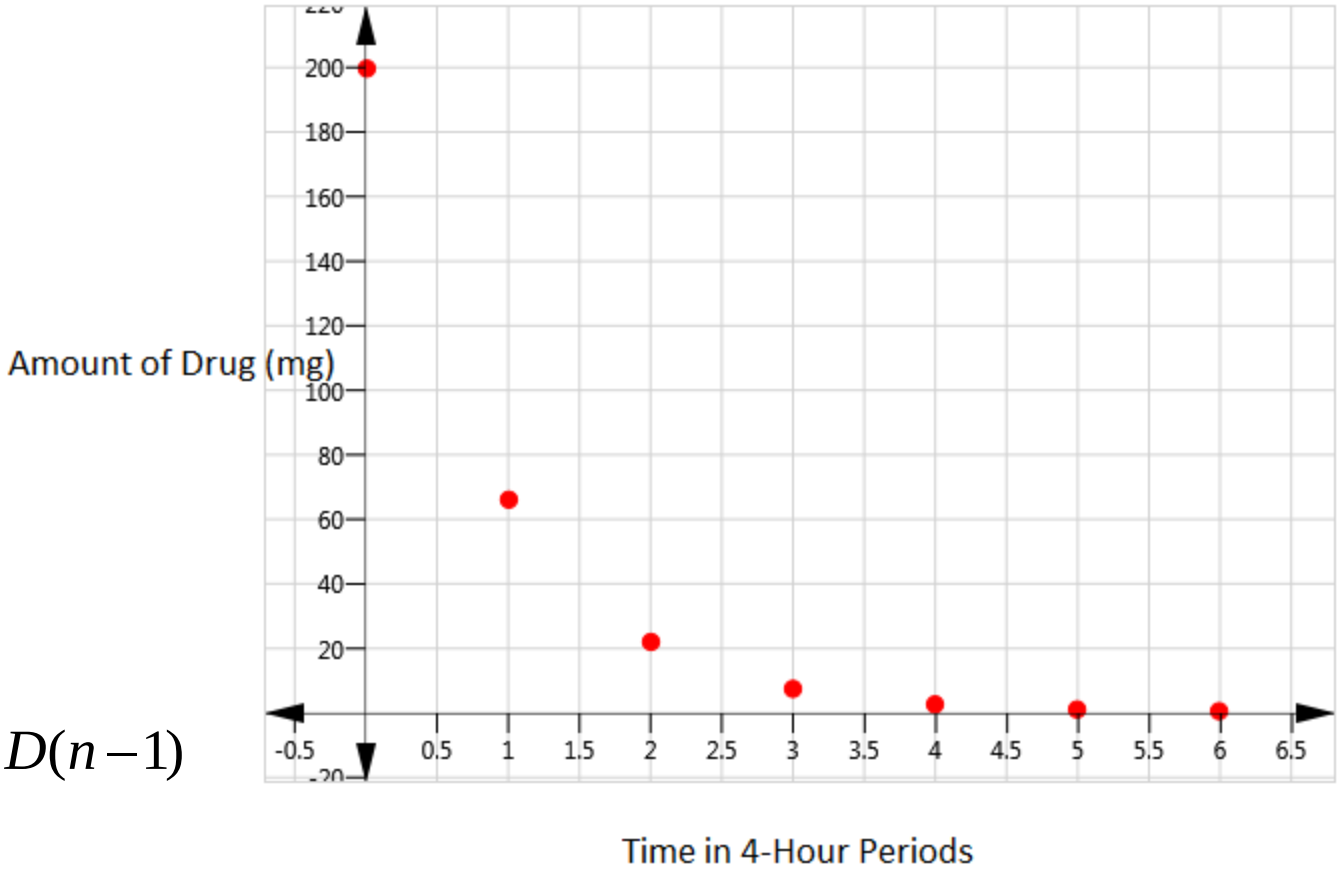
Use the calculator simulator…

Consider Multiple Representations

- Algebraic – Recursive equations
- Numerical – Table of values
- Graphical – Think about a setting an appropriate window for the context of the problem.

Multiple Representations

n	Drug (mg)
0	200
1	66
2	21.780
3	7.187
4	2.372
5	0.783
6	0.258



$$D(0) = 200$$

$$D(n) = D(n-1) - 0.67\ D(n-1)$$

Building a Closed Form Function

$$D_0 = 200$$

$$D_1 = D_0 - 0.67D_0 = 0.33D_0$$

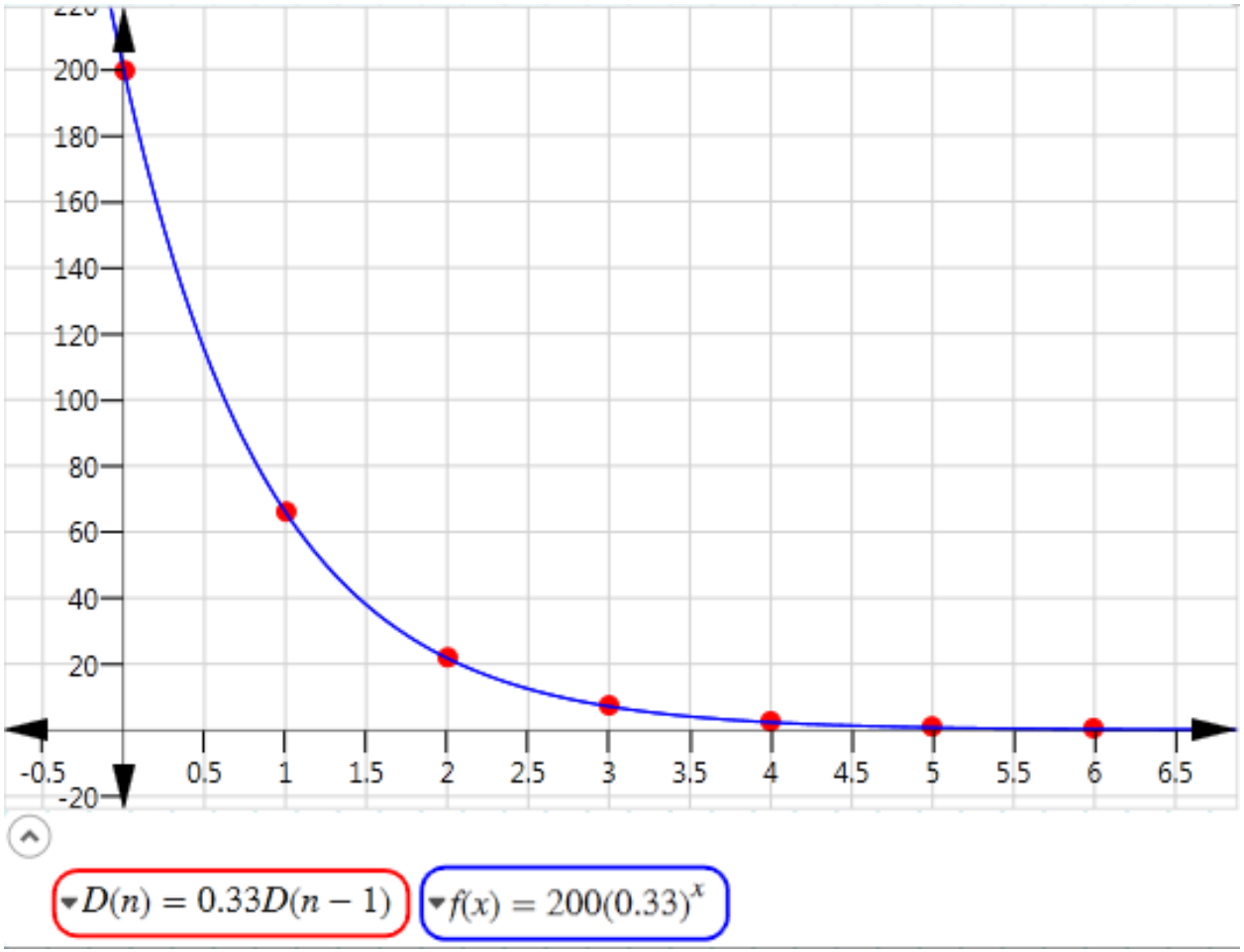
$$D_2 = D_1 - 0.67D_1 = 0.33D_1 = 0.33(0.33D_0) = 0.33^2 D_0$$

$$D_3 = 0.33D_2 = 0.33^3 D_0$$

$$\vdots$$

$$D_n = 0.33^n D_0$$

Compare Closed Form to Recursive Model



Example 2: Balance in an Account

Suppose that you deposit $1200 into an account that earns 4% annual interest and that the account pays you the interest at the end of each year.

How much will you have in the account after 10 years?

Write a set of recursive equations to model the balance of the account after **n** years. Then write a closed form function for the balance of the account **t** years after the money is deposited.

We can see how repeated multiplication can help us build an exponential model.

$$B_0 = 1200$$

$$B_n = B_{n-1} + 0.04B_{n-1}$$

or

$$B_n = 1.04B_{n-1}$$

$$B(t) = 1200\left(1.04^t\right)$$

We can think about the advantages of using each model.

Use each to answer the question: How much will be in the account after 10 years?

Pose some other questions and discuss which model would be easier to use to answer these questions.

Example 3: Fish and Wildlife

The Fish and Wildlife Division monitors the trout population in a stream that is under its jurisdiction. Its research indicates that natural predators, together with pollution and fishing, are causing the trout population to decrease at a rate of 20% per month. The Division proposes to introduce additional trout each month to replenish the stream. Assume the current population is 300. Use recursive equations, tables and graphs to investigate the following questions.

Example 3: Fish and Wildlife

1. What will happen to the trout population over the next 10 months with no replenishment program?

2. What is the long-term result of introducing 100 trout into the stream each month?

Ties to the Candy Problem and the idea of equilibrium.

Recursion Part II

In the next webinar we will extend what we learned in Part I. Exploring real world problems such as the payment of a loan or how pollution moves through The Great Lakes, we will build the recursive equations and explore when a system has an equilibrium value.

Stay Tuned
Tuesday, December 4
3:45 – 4:45

Using our students ideas to create the mathematics…

Reflect on the ideas we have discussed and consider how we have asked students to be part of the process in developing the mathematical concepts.

Thank You!

RESOURCES

- Links for NCSSM Recursion Materials

http://www.dlt.ncssm.edu/stem/content/lesson-1-introduction-recursion

- Links for DPI Math Resources (HS Math Wiki)

http://maccss.ncdpi.wikispaces.net/High+School

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