



# Advanced Learners and the new NC SCOS

Advanced Math and Science Content  
for Teachers of Advanced Learners

## MATH SESSION # 1

Using Data to Explore Linear Functions

Grades 7 - 12

Date: October 23, 2012

Developed in partnership with DPI~AIG and  
NC School of Science and Mathematics

# Advanced Content for Teachers of Advanced Learners

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- Why?
  - To ensure the growth of advanced learners
  - To develop teachers' understanding of advanced math/science content and instructional practices
- What?
  - 14 Content-based PD sessions, webinar and archived; 7 mathematics, 7 science

# Using Data To Explore Linear Functions

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Goals: To explore the topic of creating linear models for real-world data. The webinar's content is focused on several Common Core State Standards in Math that are listed at the end of the presentation.

We will interpret the constants in our model in context and assess the goodness of fit using the concept of residuals.

# As we work through the problems:



- Consider how your student will approach the problem.
- Anticipate their responses and questions.
- Consider how we can use their ideas and contributions to build the mathematical concepts.



## Example 1: Nutrition and Hamburgers

The data shown on the next slide gives the calories, amount of fat (in grams) and the amount of carbohydrates (in grams) in various burgers.

a. Using your calculator, create a scatter plot with fat content on the horizontal axis and number of calories on the vertical axis.



# Data from NCTM Core Math Tools

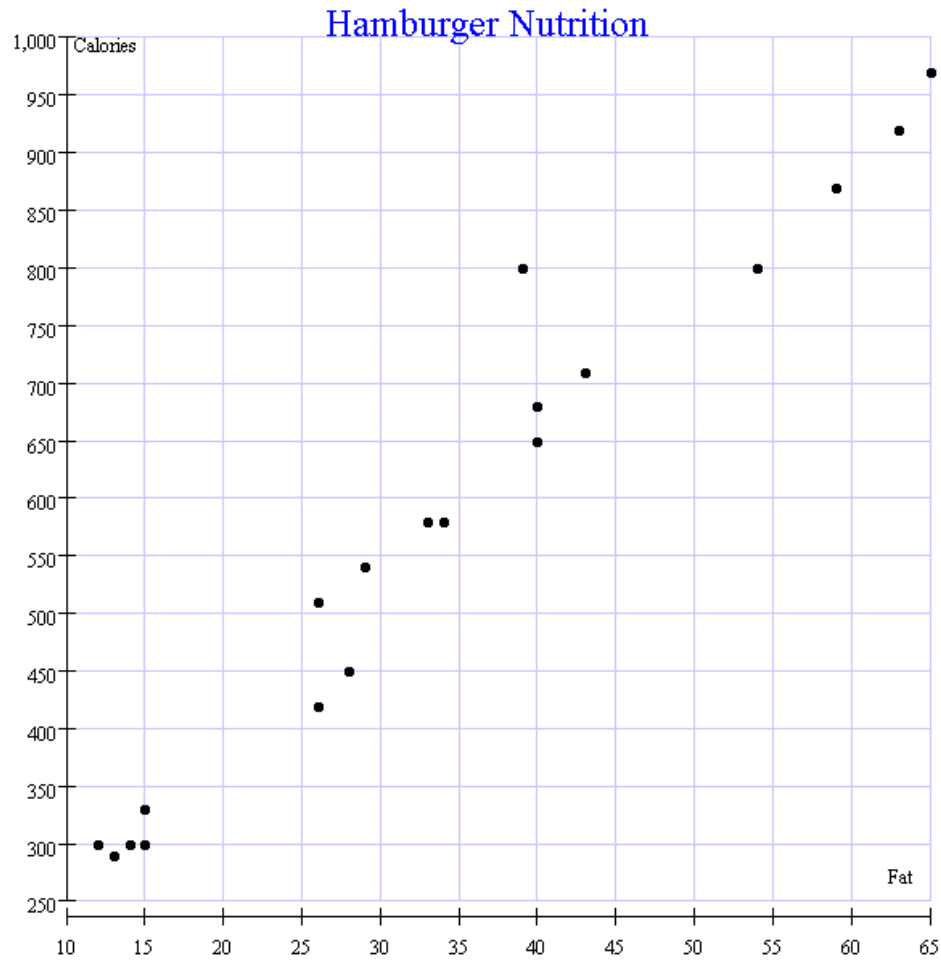


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Calories	Fat	Carbs
300	12	33
510	26	40
800	39	66
540	29	45
300	14	28
420	26	30
710	43	52
650	40	40
290	13	26
580	33	42
300	15	24
680	40	42
800	54	44
450	28	33
710	43	44
870	59	45
330	15	32
580	34	45
920	63	45
970	65	52

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# Find a Linear Model Using Your Knowledge of Linear Functions



# Comparing Models

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- Compare your model to your neighbor's model.
- Who has a “better” model? Explain.
- List some ideas for measuring goodness of fit:



# After we have decided on a criterion for goodness of fit...



- Interpret the constants in your model in the context of the problem.
- What is the 'x'-intercept for your model? Does the 'x'-intercept have some interpretation in the context?
- How can we use our models to make predictions?

# Creating a Residual Plot

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- How do we calculate a residual for a single data point?
- How do we calculate the residuals for the entire data set?
- What information can we get from a residual plot?

## Example 2: Voters



The data shown on the next slide gives the number of votes cast in presidential elections in the US in various years since 1840.

a. Using your calculator, create a scatter plot with years since 1840 on the horizontal axis and number of votes cast on the vertical axis.

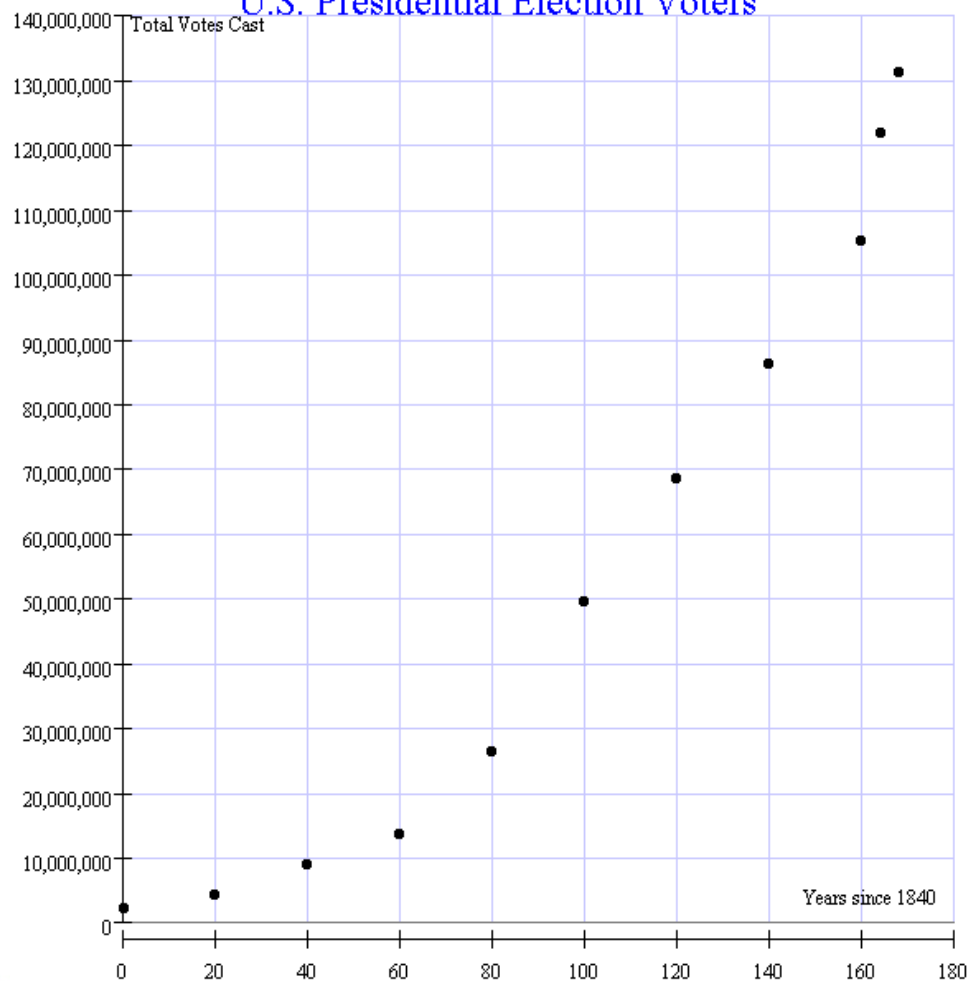
# Votes cast in US presidential elections in years since 1840



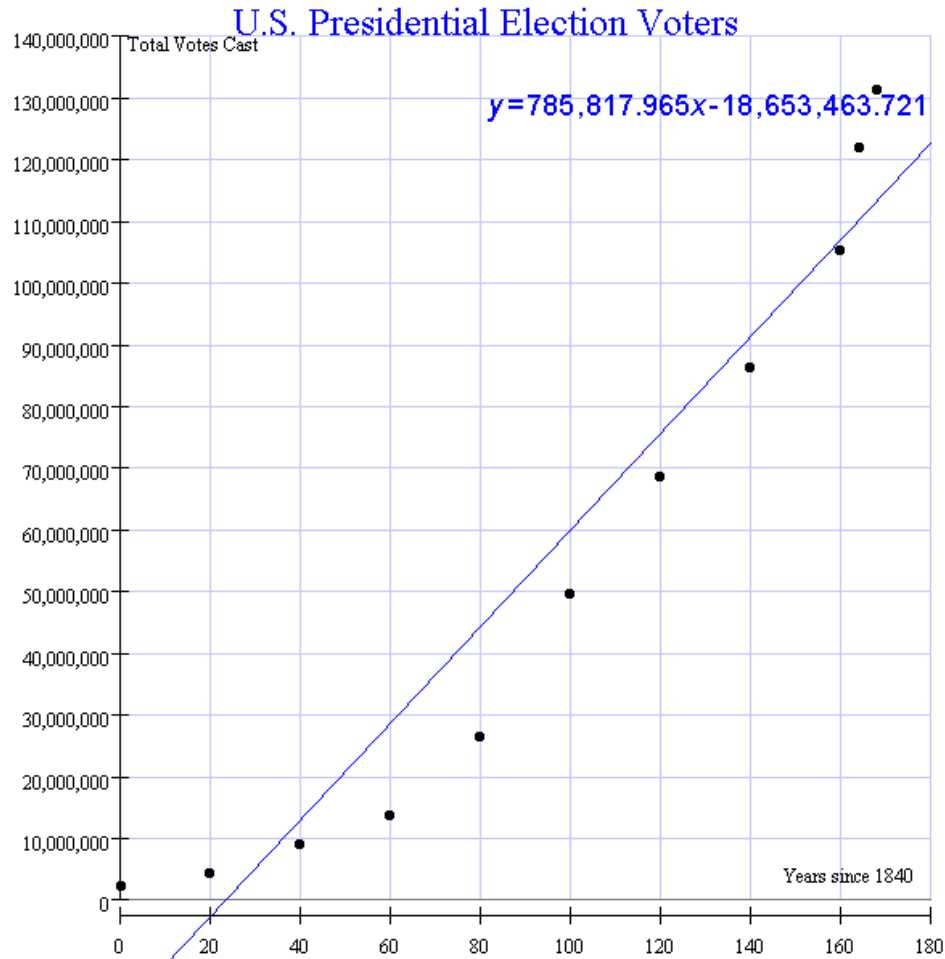
Yr since 1840	# Votes Cast
0	2,412,694
20	4,681,267
40	9,217,410
60	13,997,429
80	26,765,180
100	49,902,113
120	68,832,482
140	86,509,678
160	105,417,475
164	122,293,548
168	131,463,122



## U.S. Presidential Election Voters



# Is a linear model appropriate for this data set?



## Using Residuals to Determine Goodness of Fit

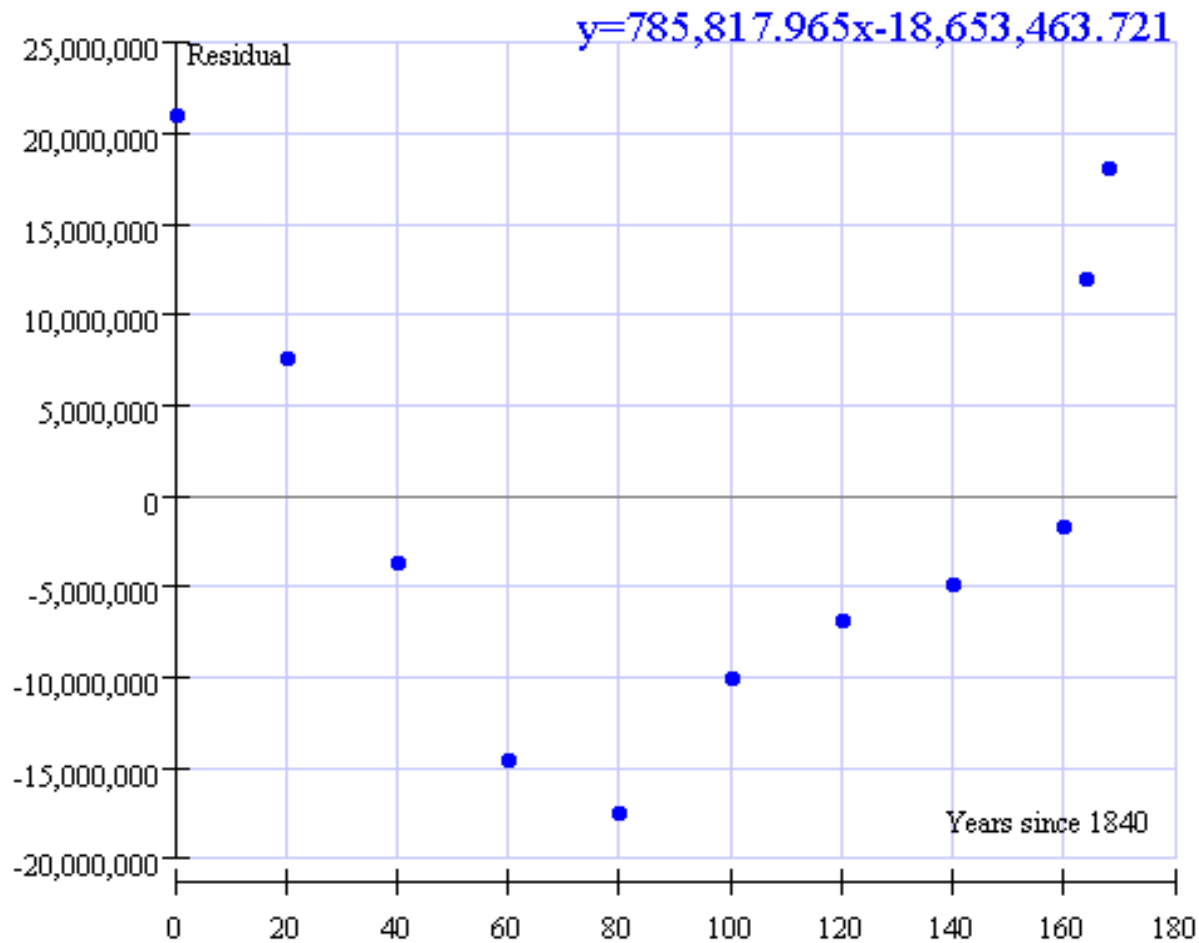


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Calculate the residuals for the entire data set and the Linear Regression model. Sketch a graph of the residual plot on your papers.

How can you use the residual plot to explain whether the Linear Regression model is an appropriate model for the data?

# Residual Plot





## How does this webinar tie to The Common Core State Standards for Mathematics?



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**Statistics and Probability: Interpreting Categorical and Quantitative Data** *Summarize, represent, and interpret data on two categorical and quantitative variables*

**S.ID.6** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear and exponential models.*

b. Informally assess the fit of a function by plotting and analyzing residuals.

c. Fit a linear function for scatter plots that suggest a linear association.

**Statistics and Probability: Interpreting Categorical and Quantitative Data** *Interpret linear models*

**S-ID.7** Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

# More Content Standards...



- **Functions: Linear, Quadratic, and Exponential Models**
- **F-LE.5** Interpret the parameters in a linear or exponential function in terms of a context.
- **Functions: Interpreting Functions** *Understand the concept of a function and use function notation*
- **F-IF.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- **F-IF.2** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
- **F-IF.9** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
- **Functions: Building Functions** *Build a function that models a relationship between two quantities*

# CCSS Mathematical Practices



- Make sense of problems and persevere in solving them
- Reason abstractly and quantitatively
- Construct viable arguments and critique the reasoning of others
- Model with mathematics
- Use appropriate tools strategically
- Attend to precision
- Look for and make use of structure
- Look for and express regularity in repeated reasoning

# Possible Extension: Error Bounds for Linear Models

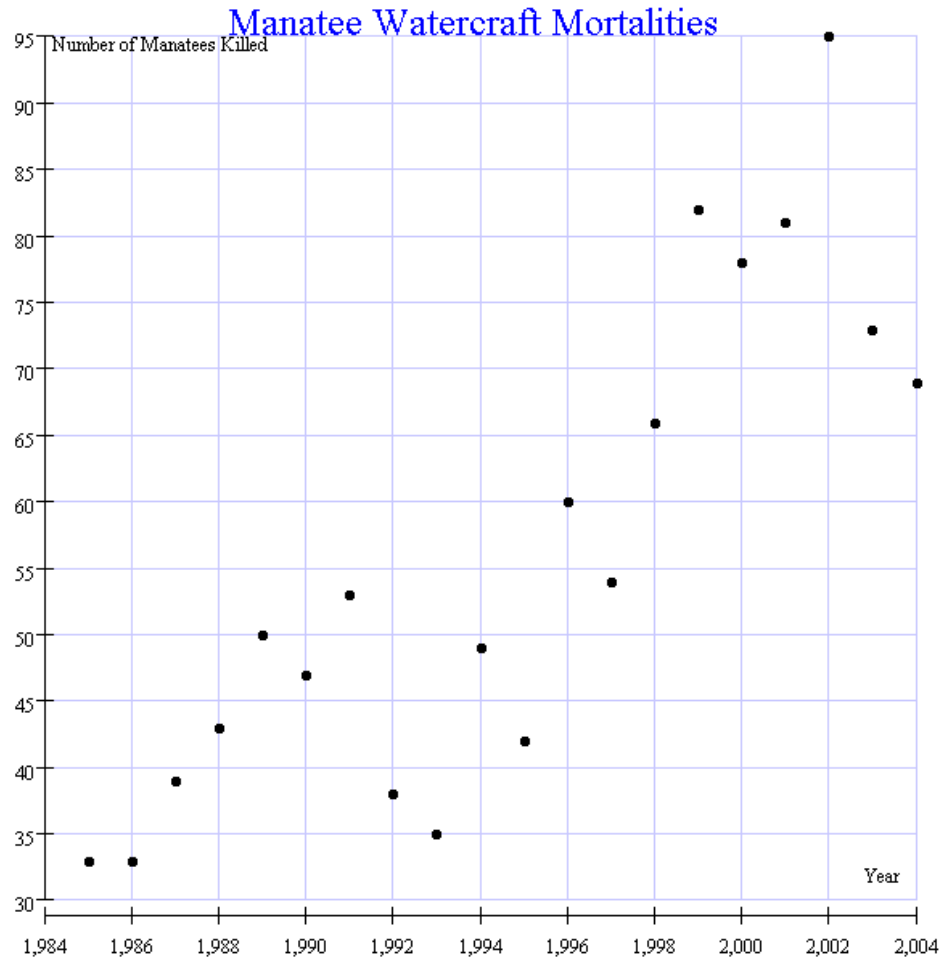


Data Sheet 1		Manatee Watercraft Mortalities	
	Year	Number of Manatees Killed	
1	1985	33	
2	1986	33	
3	1987	39	
4	1988	43	
5	1989	50	
6	1990	47	
7	1991	53	
8	1992	38	
9	1993	35	
10	1994	49	
11	1995	42	
12	1996	60	
13	1997	54	
14	1998	66	
15	1999	82	
16	2000	78	
17	2001	81	
18	2002	95	
19	2003	73	

## Manatee Watercraft Mortalities

A manatee is a large sea mammal native to Florida waters that is listed as endangered. The chart above gives the number of manatees killed in watercraft collisions near the Gulf Coast of Florida every year from 1985 through 2004. (Source: [www.savethemanatee.org/mortalitychart.htm](http://www.savethemanatee.org/mortalitychart.htm))

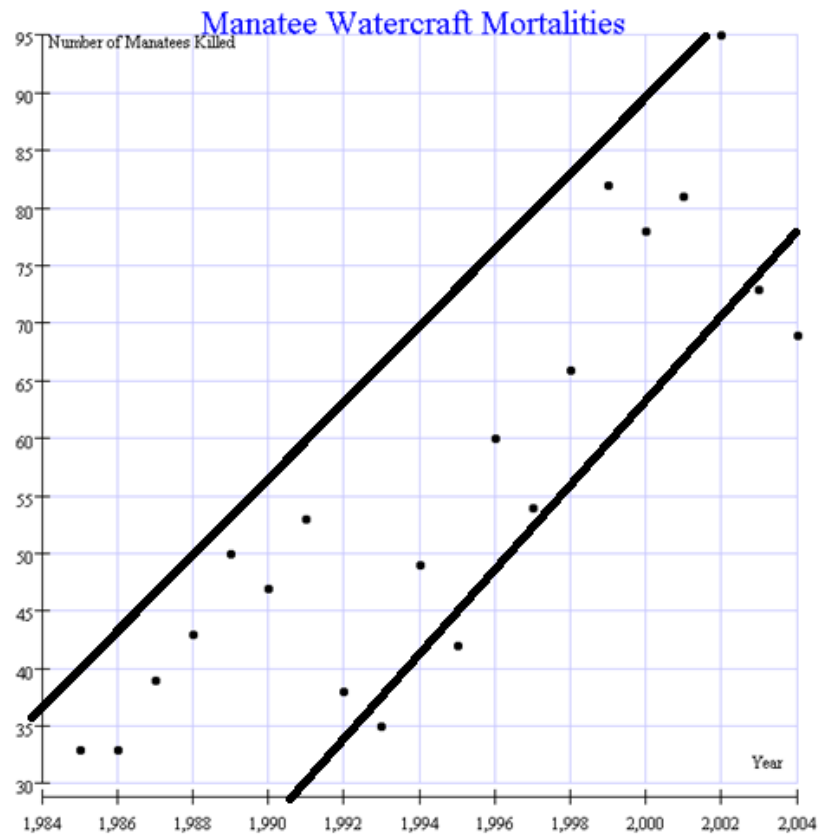
# Scatter Plot



# Instead of a single linear model



We would like a band to represent the data.



What are the advantages of having this band?



- 
- How do we find a *reasonable* band?

Ideas?



**Using our students ideas to create  
the mathematics...**

**Reflect on the ideas we have  
discussed and consider how we  
have asked students to be part of  
the process in developing the  
mathematical concepts.**

**Thank You!**





# RESOURCES



- Links for NCSSM Materials  
Algebra II and AFM – Look for Linear Data  
<http://www.dlt.ncssm.edu/algebra/curriculum.htm>  
<http://www.dlt.ncssm.edu/AFM/>
- Links for DPI Math Resources  
(HS Math Wiki)  
<http://maccss.ncdpi.wikispaces.net/High+School>
- Sneha Shah-Coltrane  
DPI Director of Gifted Education and Advanced Programs  
919-807-3849 [Sneha.shahcoltrane@dpi.nc.gov](mailto:Sneha.shahcoltrane@dpi.nc.gov)

Data sets came from NCTM Math Core Tools site.